Exercise - Digital image coding with DPCM systems

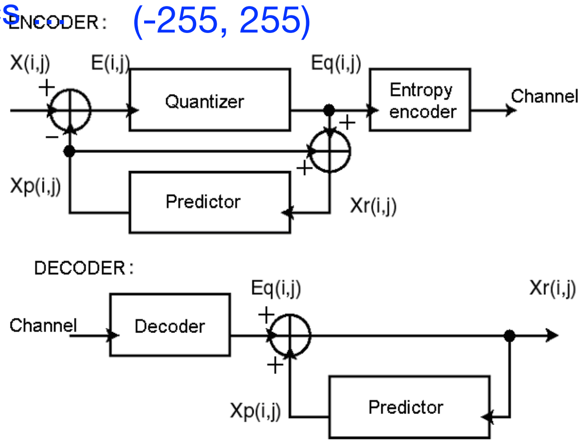
In this exercise, no programming is required. The goal is to perform a DPCM system for a small still grayscale image.

DPCM Encoder

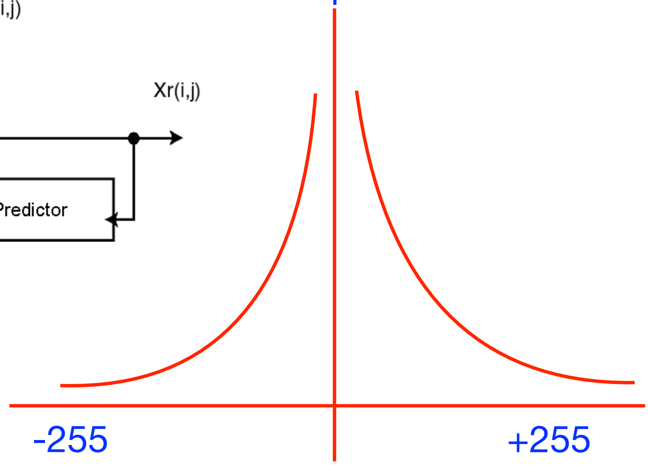
The scheme of the DCPM system is displayed on the figure below:

Consider the signals dynamics (-255, 255) E = X - Xp

(0, 255)



Repartition of the E



Where:

- X(i,j): pixel to encode;
- Xp(i,j): prediction of the pixel ;
- E(i,j): prediction error;
- Eq(i,j): quantized prediction error;
- Xr(i,j) : the reconstructed pixel (after decoding), the Xr(i,j) values are used for the next predictions.

The image is scanned with a raster scan. The notation of the pixels used for the prediction is given on the figure below (where X is the pixel to be predicted):

B	C
A	**X**

Here we consider that the predictive function is P(X) = A, and we will assume that the first column is transmitted by another process without distortion in order to initialize the system.

The quantization law used for quantizing the prediction error is given below:

E(i, j)	Eq(i, j)
] -255, -113]	-126
] -113, -89]	-101
] -89, -66]	-77
] -66, -48]	-55
] -48, -28]	-37
] -28, -13]	-19
] -13, -4]	-7
] -4, 2]	-2
] 2, 5]	3
] 5, 14]	8
] 14, 29]	20
] 29, 47]	38
] 47, 67]	56
] 67, 90]	78
] 90, 114]	102
] 114, 255]	127

1. The coding error is equal to X(i,j) - Xr(i,j). Show that this error is equal to E(i,j) - Eq(i,j). Explain what this means?

2. Let us consider that the encoder before the transmission channel uses a binary natural code (fixed length code). How many bits are necessary to transmit the Eq(i,j) values? Explain.

3. Fill in the tables of the next page. Each table represents an image, respectively X, Xp, E, Eq and Xr.

4. In case of an adaptative DPCM coder, the predictive function would be:

$$\begin{cases} P(X) = A & if \quad |B-C| < |B-A| \\ P(X) = C & if \quad |B-C| > |B-A| \end{cases}$$

   Explain how this adaptive predictive function works (consider the case of a vertical transition then the case of a horizontal transition in the image).

Tables relating to question 3

X

12	20	35	50	55	60
26	38	59	80	82	90
120	125	152	156	160	162
152	125	118	110	100	30

Xp

	12	20			

E

	8	15			

Eq

	8	20			

Xr

12	20	40			
26					
120					
152					

Note: here the first row and column index is equal to ‘1’.

Solution to the exercise: Digital image coding with DPCM systems

1. The definition of the DPCM coding error ε is:

$$\varepsilon = X(i,j) - Xr(i,j)$$

By reading the previous scheme of DPCM systems:

- $Xr(i,j) = Xp(i,j) + Eq(i,j)$
- then: $\varepsilon = [X(i,j) - Xp(i,j)] - Eq(i,j) = E(i,j) - Eq(i,j)$

The only coding error is a quantization error.

2. The prediction error is quantized with 16 reconstruction levels. By using a fixed length code, one needs **4 bits per symbol** for transmitting the Eq(i,j) values.

3. By using the predictive function P(X) = A we cannot predict the pixels of the first column of the original image. We thus assume that this first column is transmitted without distortion by another method.

The predictive function is defined by P(X) = A, therefore for any pixel with coordinates (i,j) so that j > 1: **Xp(i,j) = Xr(i,j-1)**.

For example if we consider the pixel with coordinates (1,3):

- $Xp(1,3) = Xr(1,2) = 20$
- $E(1,3) = X(1,3) - Xp(1,3) = 15$
- $Eq(1,3) = Q[\ E(1,3)\ ] = Q[\ 15\ ] = 20$
- $Xr(1,3) = Eq(1,3) + Xp(1,3) = 20 + 20 = 40$

If we reiterate the same operations for all the pixels, we will obtain the following tables:

- *Image Xp :*

X	12	20	40	48	56
X	26	34	54	74	82
X	120	123	143	151	159
X	152	133	114	107	100

- *Image E :*

X	8	15	10	7	4
X	12	25	26	8	8
X	5	29	13	9	3
X	-27	-15	-4	-7	-70

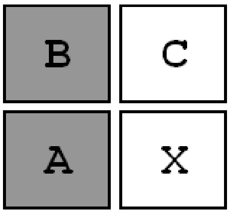
- *Image Eq :*

X	8	20	8	8	3
X	8	20	20	8	8
X	3	20	8	8	3
X	-19	-19	-7	-7	-77

- *Image Xr :*

12	20	40	48	56	59
26	34	54	74	82	90
120	123	143	151	159	162
152	133	114	107	100	23

4. The adaptive DPCM system allows you to adapt the prediction according to the contours. Let us consider that there is a strong horizontal gradient of luminance (vertical contour) in the image:



Then an adapted prediction must predict X with the pixel C value instead of the pixel values A or B. In the same way, if the contour is horizontal, an adapted prediction must predict X with the pixel A value instead of the pixel values B or C. The predictive function that allows you to perform this prediction is defined by:

$$\begin{cases} P(X) = A & \text{if} \quad |B-C| < |B-A| \\ P(X) = C & \text{if} \quad |B-C| > |B-A| \end{cases}$$